Attempts to Develop the Theory of Superfluid Turbulence. Dynamics of Single Lines vs. Kinetics of Many Loops

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Importance of Analytical Studies

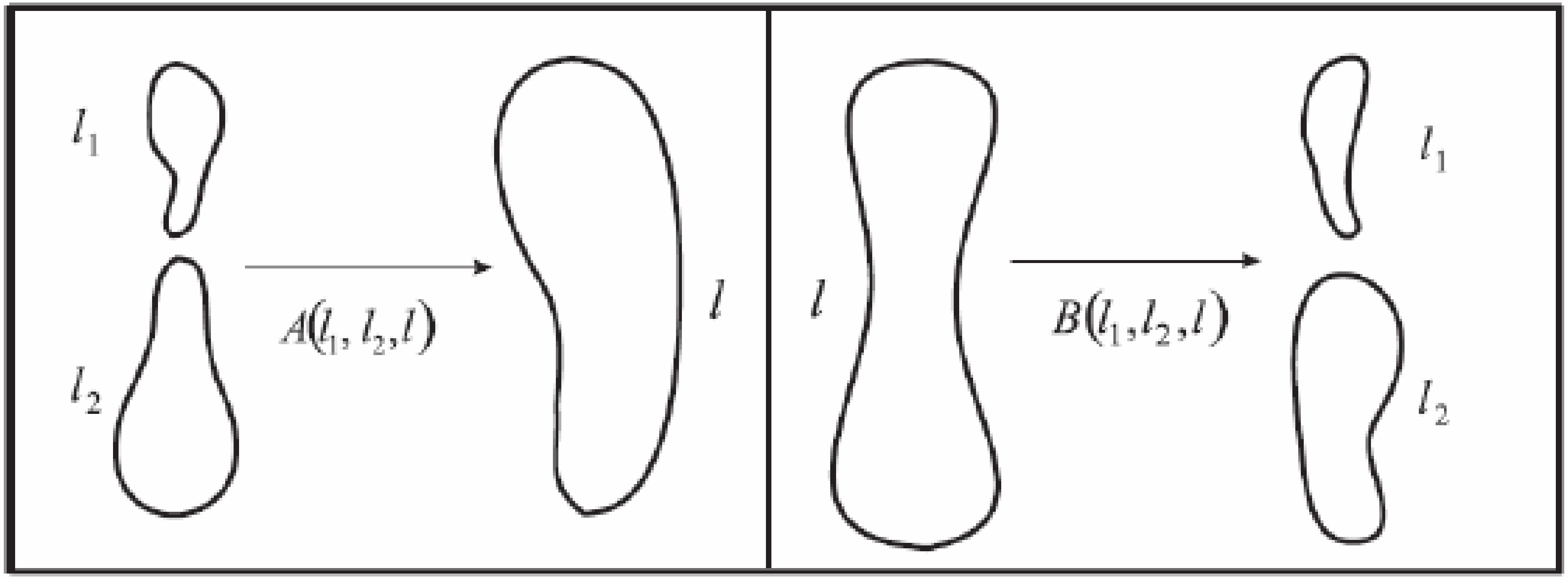
- The main part of our notion about the vortex tangles structure and dynamics came from the direct numerical simulations (and from the experiment of course). The theory is far behind. It is not good, and not only it is not good for the theory, it is also not good for other branches. Indeed, in order to make more efficient and reliable both the numerical study and experimental measurements, the support and new ideas from the analytical investigations are obviously required. Moreover, that the both experiment and numerical simulations encounter a series of difficulties

The scarcity of analytic investigations is related to the incredible complexity of the problem. In general, the dynamics of the vortex tangle consists of two main ingredients.

The first one (deterministic) is the motion of the elements of lines, due to equations of the motion (Biot-Savart law, mutual friction etc.).

The second one is the (random) collisions (and merging), or self-intersections (and splitting) of the vortex loops.

The number of reconnections is huge, therefore the dynamics of the superfluid turbulence can be thought as a kinetics of the merging and splitting vortex loops which, in addition, posses the own very involved dynamics with infinite number degrees of freedom. The number of these loops is arbitrary and



Full statement and possible simplifications

- **Thus, in the full statement of the problem we have to deal with a set of objects (vortex loops), which do not have a fixed number of elements, they can be born and die. Thus, some analog of the secondary quantization is required with the difference that the objects themselves possess an infinite number of degrees of freedom with very involved dynamics. Clearly, this problem can be hardly resolved in the nearest future. Some approaches crucially reducing the number of degrees of freedom are required.**
- **The first ones deals with dynamics of single line. Reconnections in these approaches are fully ignored (Migdal, Svistunov et al., L'vov et al. Kelvin waves, large deformations, Schwarz kinetic equation). In the better cases authors mentioned about the important role of reconnections, however, it is unclear how to introduce them in the theoretical scheme.**
- **The other, in a certain sense, opposite approach deals with the kinetics of the merging and splitting vortex loops, paying little attention to their own dynamics (Williams, Copeland et al.,Edwards, Nemirovskii,).**
- **There are some other (more exotic) approaches, such as the percolation theory (Schakel, Chorin), or drawing some properties of the vortex tangles from theory of fractals or from topological and geometric considerations (Barenghi et al), but here we will concenrate on the first two ingredients.**

Studies on the the single line dynamics

- Studies on the the single line dynamics (ignoring reconnections), can be useful for some particular situations (e.g. Kelvin waves cascade is able to explain the low temperature decay). However this way to develop the whole theory of the superfluid turbulence is the dead-end way, just because it does not allow to incorporate the reconnections at all.

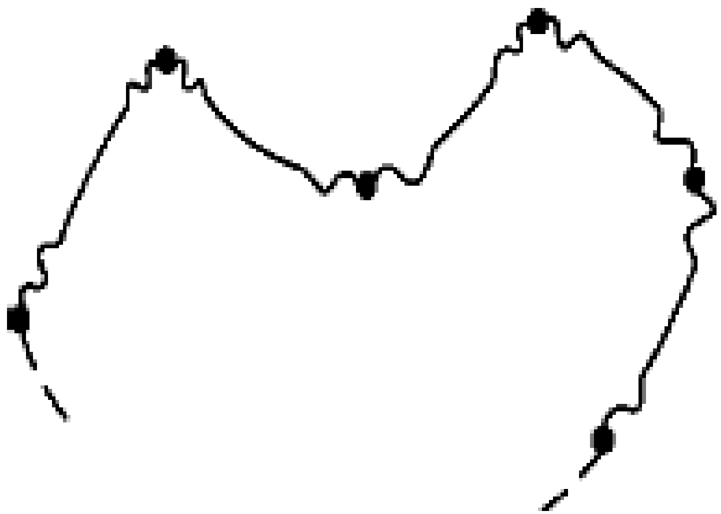
Why it is impossible to introduce reconnections

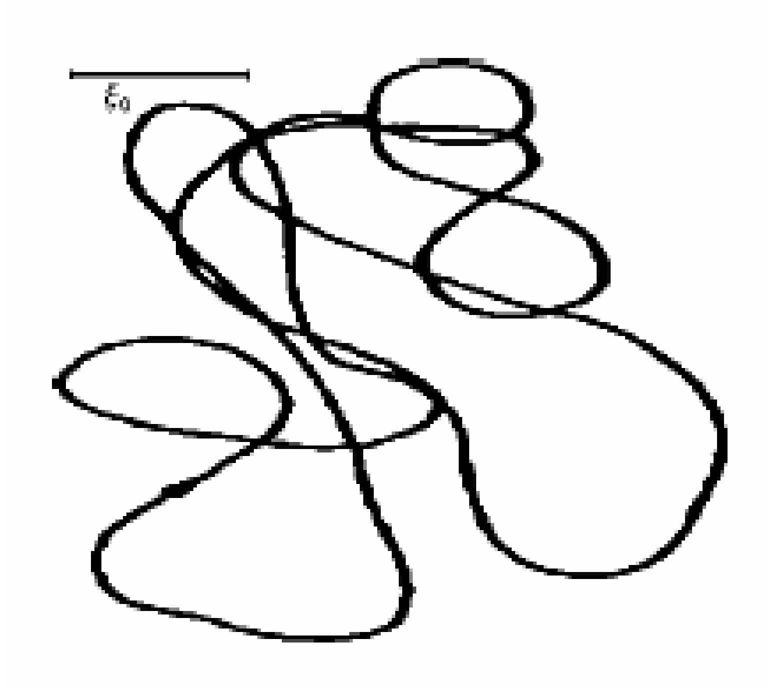
As stated the approaches of the first kind start from the equation of motion of line

$$ds(\xi)/dt=F(\{s(\xi,t)\},\xi,t)$$

Here $F(\{s(\xi,t)\},\xi,t)$ is functional of the whole vortex line configurations $\{s(\xi)\}$, in general, nonlinear and nonlocal. The label variable ξ take the values $0<\xi<L$, where L is the length of the loop. The evolution, described by this equation has a sense only up to moment, when the line either collides and reconnect with other loop forming the larger loop, or the self-reconnection occurs and two daughter loops appear. In this case the some new set of the variable ξ should be inserted inside interval $0<\xi<L$ or, on the contrary removed from it. What is said, just a general description, it is not clear how it can be realized in analytical form, moreover the number of the reconnections is huge

Brownian Vortex Loop





$$P(\{\mathbf{s}(\xi,t)\}) \propto \exp\left(-\int_0^l \int_0^l \mathbf{s}'^{\alpha}(\xi,t)\Lambda_{\alpha\beta}(\xi,\xi')\mathbf{s}'^{\beta}(\xi',t)\,d\xi\, d\xi'\right)$$

The theory of superfluid turbulence as the kinetics of interacting (colliding) vortex loops

1. the distribution function $n(l) \propto l^{-5/2}$ of number of loops having the length l.

2. The full rate of the reconnections. $\dot{N}_{rec} = C_{rec} \kappa L^{5/2}$ was derived

3. The flux of the length (and energy) $\dot{L} = -C_{decay} \kappa L^2$ due to the Feynman cascade like breakdown of loop was evaluated

4. The famous Vinen equation for the evolution of quantity $L(t)$ was derived and its properties in terms of the merging and splitting loops had been discussed.

5. The diffusion like evolution of the nonuniform vortex tangle was described.

Summary

We listed the results obtained, just to assert, that even very simple approach (Boltzman type equation for density of Brownian loops) is in position to describe many features of quantum turbulence. But, the more important conclusion is that this consideration is (to my knowledge) the only attempt allowing to include the reconnection processes into theory describing the superfluid turbulence. In principle, there are no obstacles (at least, fundamental, not technical) to add other more subtle degrees of freedom (e.g. torsion) to describe the "inner dynamics" of loops in more details. It is possible also to modify statistics of loop to describe other physical cases. For instance it is possible to describe strongly the strongly anisotropic, polarized bundles of line, which likely appear in quasiclassic turbulence of quantum fluids.

As a summary it can be stated, that the futre theory describing the both structure and dynamics of the vortex tangles, should be ground on the consideration of the vortex tangles as a set of colliding vortex loops with few degrees of the freedom.