The one-fluid model of helium II based on the Extended Irreversible Thermodynamics

Michele Sciacca

Università di Palermo

Abu Dhabi, 2–5 May 2011

The main ideas beyond EIT

- Jou D., Casas-Vázquez J, Lebon G., (2001), *Extended Irreversible Thermodynamics*, Springer-Verlag, Berlin.
- Liu, I (1972), *Method of Lagrange multipliers for exploitation of entropy principle*, Arch.Rat. Mech. Anal., Vol. 46, 131-148
- Muller I, Ruggeri T., (1993), *Extended Thermodynamics*, Springer-Verlag, Berlin.

In EIT, the space $V = C \bigcup F$, where C being classical variables (mass, momentum, energy, and composition), and F the corresponding fluxes (they are fast and non-conserved)
What does EIT require?

- Choose the right variables V for the system;
- Write general balance equations for the basic variables a:

$$\rho(da/dt) = -\nabla \cdot \mathbf{J} + \sigma$$

- Determine the constitutive equations for the fluxes , deduced from the second law of thermodynamics, using the Liu method of Lagrange multipliers. Note that the entropy is function of all the variable V, and it is a concave function of the state variables, and its rate of production is locally positive:

$$\rho(ds/dt) + \nabla \cdot J^s = \sigma^s \geq 0$$

The monofluid model in turbulent helium II

In the equilibrium Thermodynamics the Gibbs equation is:

$$Tds = du + pd\left(\frac{1}{\rho}\right)$$

(using this expression one can infer the Navier Stokes equation for the laminar case.) The Extended Irreversible Thermodynamics (EIT) requires the presence of the heat flux in the variables of the our system

$$Tds = du + pd\left(\frac{1}{\rho}\right) + \frac{\alpha_1}{\rho}\mathbf{q}\cdot d\mathbf{q}$$

Applying EIT to this helium II (because the extremely high thermal conductivity), and neglecting dissipative phenomena, one finds

$$\begin{cases} d\rho/dt + \rho\nabla\cdot\mathbf{v} = 0 \\ \rho(d\mathbf{v}/dt) + \nabla p^* = 0 \\ \rho(du/dt) + \nabla\cdot\mathbf{q} + p\nabla\cdot\mathbf{v} = 0 \\ d\mathbf{q}/dt + \zeta\nabla T = 0 \end{cases} \tag{1}$$

- This model proves the existence of the second sound.
- Note that the Gibbs equation is equal to the one considered by Landau:

$$Tds = du + pd\left(\frac{1}{\rho}\right) + \frac{\rho_n}{2\rho_s}d\left(\mathbf{v}_n - \mathbf{v}_s\right)^2$$

- The presence of vortex tangle can be modeled by a tensor σ_q in the right hand side of the last equation (it is strictly connected to the mutual friction).
- One-fluid-two fluid connection: $\mathbf{q} = \rho_s T s \mathbf{V}_{ns}$ and $\rho v = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$.

Q: What more do we find inside the EIT?

A: One can choose other fields as independent variable, as for instance the vortex line density L.

Neglecting the nonlinear terms in the fluxes of the independent variables, we get

$$\begin{cases} d\rho/dt + \rho\nabla\cdot\mathbf{v} = 0 \\ \rho d\mathbf{v}/dt + \nabla p^* = 0 \\ \rho du/dt + \nabla\cdot\mathbf{q} + p\nabla\cdot\mathbf{v} = 0 \\ d\mathbf{q}/dt + \zeta\nabla T + \chi\nabla L = \sigma_q = -\kappa B/3L\mathbf{q} \\ dL/dt + L\nabla\cdot\mathbf{v} + \nabla\cdot(\nu\mathbf{q}) = \sigma_L = -\beta_q L^2 + \alpha_q|\mathbf{q}|L^{3/2} \end{cases} \tag{2}$$

In steady state, from the last two equations one find:

$$\mathbf{q} = -\frac{3\chi}{\kappa BL}\nabla L - \frac{3\zeta}{\kappa BL}\nabla T$$

and $J^L = \nu\mathbf{q}$.

- This model, when high frequency is considered, shows vortex density waves.
- σ_q and σ_L depend on the experiment, so in general they depend on anisotropy and polarization (and one can give evolution equations for them)

Some References:

- Jou, D., Lebon, G. Mongiovi' M.S. (2002), Phys. Rev. B, Vol. 66, 224509.
- Mongiovi', M.S. (1991), J. Non-Equilib. Thermod., Vol. 16, 225.
- Mongiovi', M.S. (1993), Phys. Rev. B, Vol. 48, 6276.
- Jou D., Mongiovi' M.S., Sciacca M., Ardizzone L, Gaeta G. *Hydrodynamical models of superfluid turbulence*. In: *Thermodynamics*, Tadashi Mizutani (Ed.) p. 233-274, INTECH, 2010.