THE UNFINISHED GAME

Pascal, Fermat, and the Seventeenth-Century Letter that Made the World Modern

A Tale of How Mathematics Is Really Done

KEITH DEVLIN

Author of The Math Gene and The Millennium Problems
Monsieur

If I undertake to make a point with a single die in eight throws, and if we agree after the money is put at stake, that I shall not cast the first throw, it is necessary by my theory that I take 1/6 of the total sum to he impartial because of the aforesaid first throw.

And if we agree after that that I shall not play the second throw, I should, for my share, take the sixth of the remainder that is 5/36 of the total.

If, after that, we agree that I shall not play the third throw, I should to recoup myself, take 1/6 of the remainder which is 25/216 of the total.

And if subsequently, we agree again that I shall not cast the fourth throw, I should take 1/6 of the remainder or 125/1296 of the total, and I agree with you that that is the value of the fourth throw supposing that one has already made the preceding plays.

But you proposed in the last example in your letter (I quote your very terms) that if I undertake to find the six in eight throws and if I have thrown three times without getting it, and if my opponent proposes that I should not play the fourth time, and if he wishes me to he justly treated, it is proper that I have 125/1296 of the entire sum of our wages.

This, however, is not true by my theory. For in this case, the three first throws having gained nothing for the player who holds the die, the total sum thus remaining at stake, he who holds the die and who agrees to not play his fourth throw should take 1/6 as his reward.

And if he has played four throws without finding the desired point and if they agree that he shall not play the fifth time, he will, nevertheless, have 1/6 of the total for his share. Since the whole sum stays in play it not only follows from the theory, but it is indeed common sense that each throw should be of equal value.

I urge you therefore (to write me) that I may know whether we agree in the theory, as I believe (we do), or whether we differ only in its application.

I am, most heartily, etc.,

Fermat.
$10 - Best of 7 coin flips

Pascal is ahead of Fermat 2-1, when the game is called on account of rain.

What is the equitable way to divide the money up?
I am a mathematics teacher and often use items from your columns as material in my classes. I have always agreed with the mathematical answers that you have given. You explain things in layman’s terms that are often better understood by my students than my mathematical proofs. But in a recent column, you gave an incomplete answer to a question posed about the fact that if 23 people are chosen at random, the probability is a bit greater than 50/50 that at least two will share a birthday. (The reader asked about the probability of 50 people.)

You should tell your readers at what point the probability becomes virtual certainty, and I'm enclosing two computer programs that I've written for you for that purpose. The one called MARLYN computes the theoretical probabilities, and BIRTHDAY simulates the actual problem.

—David Pleacher, Winchester, Va.

Thank you! I should have done this myself. The results are amazing! With only 58 people in a group, the chances are more than 99% that at least two will share the same birthday. Readers can test this themselves by asking the birthdays (month and day) of the next 58 people they meet at the mall. (You can ignore the ones who scowl at you in response; the next fellow will be just as random.) And let me know what happens!

We ask readers to compare birthdays for an experiment (but asking for ages may get you in trouble)
Pattern Recognition and Pareidolia

Faces in Things @FacesPics · May 18
Heyyy yooouuu guuuys
<table>
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<th>3</th>
<th>4</th>
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<td>6.3%</td>
<td>3.1%</td>
<td>1.6%</td>
<td>0.8%</td>
<td>0.4%</td>
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Description
Backgammon is a lot of fun with iGammon. It’s easy to move the checkers by dragging them. Or touch a checker you want to move, then the point you want to move it to. The smooth animation gives the game a great feel.

What's New in Version 4.2
Fixed bug where a checker is flipped on edge at the start of a new game.

Screenshots
COMPUTER PLAYER

The computer AI is VERY strong. There are four styles of player: Cautious, Wild, Advanced, and Expert. All of them are fun and challenging. [By the way, the dice in all levels of iGammon are random and fair. Sometimes you will get bad luck. (Yes, backgammon can be a cruel game!) Don’t let bad luck affect your game. Concentrate on doing the best you can and you will see your game improve over time.]

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Tom Keith Web Site › iGammon Support › ...More
UPDATED REVIEW: Congratulations, Tom Keith! ★★
by Adam Schramm

UPDATE: Let's give a round of applause to iGammon developer Tom Keith for winning the award for "Most Outrageous Statement By a Developer Regarding Their App" for his claim, "...the dice at all levels of iGammon are random and fair." It was a close contest between Tom; Zenessa, for all of Professor McMurry's Subliminal...More

Unfair Odds ★★
by Rmc409

The game won 48 out of 52 first rolls...Need I say more?

Rigged dice & unfair ★
by Frustrated too.

I really love the look and feel of this game – but the dice are rigged. The computer opponent conveniently rolls multiple doubles when needed and the prefect roll to knock you, double the cube and then seal you in for the kill. Frustrating waste of time.
Backgammon NJ HD
Jimmy Hu
Offers In-App Purchases

Rating: ★★★★★ (119)

Details Reviews Related

Description
- GAME CENTER -- Play online, chat, invite friends, auto-match with random players
- [ Real-time ] Play online in real-time
- [ Turn-based ] Play online at your own pace
- VERSATILE -- Play offline vs. computer or against another person in 2-player mode
- CUSTOMIZABLE -- More options than any other backgammon app... more
Backgammon NJ does not cheat

There are only two ways that a backgammon game could cheat:

1. Changing the dice rolls.
2. Using knowledge of upcoming dice rolls to its advantage.

It's easy to show that our game doesn't change the dice rolls: print out upcoming dice rolls before you play, compare the printout to the rolls in the game, and you'll see that the dice rolls match exactly.

It's easy to show that our game doesn't use knowledge of upcoming dice rolls to its advantage: compare a game played with computer-generated dice to a game where the same dice rolls are input with manual dice. On Expert level, the computer will always make the same move in the same position, regardless of upcoming dice rolls. If the game were cheating, it would play differently with computer-generated dice than with manual dice because it would be looking at upcoming dice rolls.

The following is a more detailed explanation of what we have covered above and how dice generation is handled in the game.

**The Dice Rolls**

Backgammon NJ uses the Mersenne Twister algorithm to generate random numbers. This is a well-known RNG (random number generator) that produces high quality random numbers. Like most RNGs, the algorithm starts with a Seed (a number) and then generates an unending sequence of random numbers from the Seed.
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It is important to note that the only thing that determines the dice rolls is the Seed. This is important because it shows that the rolls do not depend on the board position, difficulty level, score, or any other factor. Essentially, all the dice rolls in the game are pre-rolled and do not change once the Seed is chosen.

**Printing Out Dice Rolls**

If you print out the dice rolls and compare them to the rolls you receive in the game, you will see that they always match. This proves that the dice rolls are not being changed during the game. For example, when the computer gets a 6-6 during bearoff, you can see that the roll came from the list and not because the computer changed the dice.

Some people think that it’s strange that our game can print out rolls in advance and that this means we aren’t really generating random dice. However, all programs that use random number generators work exactly the same way. They all generate a reproducible sequence of numbers that can be printed out in advance. The only difference is that we’re showing the numbers to you to prove a point.
There are three ways of printing out dice rolls. In order of preference, they are:

1. Visit http://rolls.njsoftware.com on your desktop computer. There, you can enter the RNG Seed to obtain a printout of up to 10,000 rolls.
2. Download the source code and Windows executable for the dice generator at http://rolls.njsoftware.com, and run the program on your desktop computer.
3. Use the “Next 100 Rolls” button within Backgammon NJ. This lists the next 100 dice rolls for your current Seed. The reason this is the least preferred method is that there is no good way to print out the rolls other than by copying them to paper by hand. You can take a screenshot and/or copy the rolls onto the clipboard, but remember that if you exit Backgammon NJ, the Seed and upcoming rolls will be reset and you will need to set them back to their original values in the Settings window.

Choosing Sides

Many people believe that even though the dice are pre-rolled, the computer always gets the “good” rolls. For example, they believe that the computer gets more doubles than they do. However, although it is certainly possible for one side to get better rolls than the other, there is a 50-50 chance that the human player will get the “good” rolls. Let’s examine why this is true.

First, let’s discuss how each side gets its dice rolls in a game. When a game starts, there is an opening roll with two dice. The player gets the first die and the computer gets the second die. Whoever
First, let's discuss how each side gets its dice rolls in a game. When a game starts, there is an opening roll with two dice. The player gets the first die and the computer gets the second die. Whoever has the higher number wins the opening roll (doubles are skipped). For example, if the opening roll were 6-1, the player would win and start with a 6-1 opening roll. If the opening roll were 1-4, the computer would win and start with a 1-4 opening roll.

After the opening roll, the players alternate rolls on the dice roll list. The winner of the opening roll will get the first, third, fifth, etc rolls. The loser of the opening roll will get the second, fourth, sixth, etc rolls. Note that if the opening roll’s dice were swapped (for example 1-6 instead of 6-1), then the opposite side would win the opening roll, and each side would end up with the other side’s rolls for the game.

Now let’s suppose that during a game the computer rolled 8 doubles and the player only rolled 1 double. One might assume that the pre-rolled dice favored the computer. But if you look carefully, the opening roll was the only thing that determined which side got the good rolls. If the opening roll had been won by the opposite side, all subsequent rolls would have changed sides and the player would have gotten the 8 doubles instead of the computer. Therefore, the opening roll is the only thing that determines which side gets which rolls, and it has a 50-50 chance of going either way.

If you don’t want to leave it to chance, this game includes an option to swap opening rolls. This option reverses the dice in opening rolls, causing the opposite player to win each time, therefore swapping dice rolls between the player and the computer. Since you are in control of this option, you control which side gets which rolls. If you...
If you don't want to leave it to chance, this game includes an option to swap opening rolls. This option reverses the dice in opening rolls, causing the opposite player to win each time, therefore swapping dice rolls between the player and the computer. Since you are in control of this option, you control which side gets which rolls. If you believe that the rolls are constantly favoring the computer, simply toggle the setting and the computer's rolls will become your rolls.

**Looking Ahead at Upcoming Dice Rolls**

Note: The following section assumes that you are playing on Expert level. On other skill levels, the computer will make random moves on occasion and will not play the same way from game to game.

Some people think that because the game can print out the next 100 dice rolls, the computer is using this knowledge to cheat. This is not the case. The dice roll printout was only added to show that the rolls were not being altered. Here is how you can prove that the game does not look at upcoming dice rolls:

1. Set the skill level to Expert.
2. Start a new match.
3. Print out the upcoming dice rolls.
4. Play out a game and record all of the moves that you make.
5. Turn on Manual Rolls.
7. Manually enter the same dice rolls and make the same moves for the game you just played.

You'll find that the computer will make the same exact moves as...
level. Expert level simply picks the highest ranked move. The lower levels randomly pick suboptimal moves. The lower the level, the more often a bad move will be picked.

**Other Features**

This game includes a manual dice option that lets you play against the computer using real dice. Here is the wrong way to use this option: lose 10 in a row against the computer, get frustrated, try manual dice, go 2-8 against the computer, and conclude that the game is cheating because you're doing better now.

The correct way to use this feature is to play 15 point matches using manual dice against Expert level and see if you can win. If you can’t beat it very often (we’re pretty sure you can’t unless you’re a professional player), then ask yourself, what reason would we have to manipulate the dice if Expert level can already play so well? The answer is that we have no reason to make the game cheat.

We’ve also open sourced our dice generation code. If you examine the source code, it proves without a doubt that our dice generation is fair.

**Thank You for Your Support**

Thank you for taking the time to read through our entire explanation. If you have any comments, please don’t hesitate to contact us at:

http://contact.njsoftware.com
IMPORTANT: The computer cannot see upcoming rolls. If you exit the app and restart, the seed will change and these upcoming rolls will be reset.

RNG Seed = 1326701408, Next roll number = 0

0: 6-5 6-1 6-1 2-3 4-2 5-2 4-5 1-1 5-6 2-1
10: 6-3 5-3 1-1 1-4 6-6 2-2 3-3 1-1 4-6 5-4
20: 2-5 2-1 5-6 5-1 5-1 1-2 3-5 3-6 3-6 3-6
30: 2-6 2-2 3-4 2-1 1-1 5-3 2-3 1-4 6-5 3-6
40: 1-2 4-4 6-2 2-2 5-6 1-5 3-3 3-6 4-1 3-2
50: 4-6 3-5 6-4 6-5 1-3 2-6 2-3 5-6 6-6 6-2
60: 3-1 2-3 1-6 4-6 6-6 6-5 2-5 3-4 3-3 2-4
70: 3-2 5-6 6-2 3-3 4-6 6-2 1-6 5-5 6-1 3-4
80: 6-4 5-2 6-2 3-1 3-6 1-6 2-5 5-5 2-6 6-3
90: 5-4 6-2 3-1 4-4 2-1 3-4 2-6 2-3 2-1 5-3
/* Backgammon NJ Dice Roll Generator */
/* Download this source file and a prebuilt */
/* Windows executable at: */
/* http://www.njsoftware.com/printrolls.zip */
/* Generates the same dice sequence as Backgammon NJ, */
/* given the RNG Seed. */
/* Usage: */
/* 1) From a console/dos window, to print the rolls: */
/*    printrolls seed num_rolls */
/* 2) From a console/dos window, to save the rolls */
/*    to a file: */
/*    printrolls seed num_rolls > file.txt */
/* 3) On Windows, double click on printrolls.exe to */
/*    bring up a temporary console window. Then enter */
/*    the seed and number of rolls. To copy the */
/*    rolls from the console window to the clipboard, */
/*    use the Edit->Mark option from the menu. */

#include <stdio.h>

/* The Mersenne Twister Random Number Generator */
#define MERSENY_MAX 624

static unsigned int MersenneTwisterNums[MERSENY_MAX];
static int MersenneTwisterIndex;

/* Seed the random number generator */
void Mersenne_Srand(unsigned int seed)
4. Developers did not make this fair

⭐⭐⭐⭐⭐ X6gt - Feb 7, 2016
First off, it's amazing how many times the computer gets exactly what it needs in the dice roll to beat me. I am a pretty good player but you can't tell that from my ranking or game play against the computer. The way they set up pre rolls is dumb. Why they didn't just program the rolls to be random I'll never know. I'm irritated I paid money for this with my only pay back being this review. I urge you not to pay for this app unless the developers will offer your money back if the computer wins by luck. However I don't think it's luck at all. They get the roll they need exactly when they need it. I played the hardest level and won maybe 40% of the time. I seem to win less on the middle level. DO NOT BUY THIS APP, YOU WILL ONLY GET FRUSTRATED.

5. Definitely Distorted

⭐⭐⭐⭐⭐ macuaig - May 6, 2016
It's a real joke. This is not a neutral game, even at Easy level. The interface is nice, but it absolutely produces absurd advantages to the computer, very occasionally offering equally ridiculous wins for the player. Dice patterns repeat comically and predictably. Computer wins the opening roll 90% of the time. Computer gets exactly what it needs...and you also get what the computer needs. Computer always gets multiple doubles just as it's bearing off. Forget about getting back on the board from the bar no ma...More

6. Greatest backgammon app I've ever seen
The Tire Question
Question 3

- Front Left: 43%
- Back Left: 6%
- Front Right: 9%
- Back Right: 9%
- Other: 32%
The Most Important Probability Trick:

Flip it around!
Other Probability Fallacies:

• Overvaluing long odds, especially on positive outcomes

• Overvaluing recent experience

• Luck, as seen though “misdeal superstitions”

• Thinking dice-like objects have “memories”
Sink the first ball, select your side, shoot again.

Sink more balls, have fewer targets, can't sink final ball without risking a loss.
Dominion

Donald X. Vaccarino

Rio Grande Games
**Militia**

+2

Each other player discards down to 3 cards in his hand.

**Remodel**

Trash a card from your hand. Gain a card costing up to 2 more than the trashed card.

**Smithy**

+3 Cards
Pressing Your Luck

(Sid Sackson, 1974)
INVADERS

SETUP
Decide which player is the invaders and which one will be the Earth Troopers.
The Invader shuffles a deck of aliens face down. The Earth player begins with 2 troops.
The Invader player secretly sets the invader success percentages.

INVADERS: steadily move towards Earth / individually attack every Trooper who moves close
TROOPERS: can fly to any region each turn / work as a group against invader targets

TURN SEQUENCE
1. Invader refresh: Invader player draws 2 new invaders & places in outer space section.
2. Earth reinforcements: Earth player gets 1 new troop for each invader on the board.
3. Trooper movement: Earth player moves each trooper ANYWHERE (in any region) -
   decide which invaders to attack by placing each trooper next to any invader.
4. Invaders attack: Secretly, the invader player rolls separately against EACH trooper target.
   If the invader roll is less than the creature’s strength, destroy that trooper.
5. Surviving troopers attack: Earth player rolls ONE TIME ONLY against each invader target.
   Each trooper adds 20% to destroy the creature. Destroy creature if roll is low enough.
6. Invade! Surviving invaders advance towards the Earth by 1 section of board.

THE END
If all invaders die, the earth player wins.
If an invader moves to the Earth space, the invader player wins.